Bull. Environ. Contam. Toxicol. (1998) 60:677-684

Bulletin of Environmental Contamination and Toxicology

Statistical Analysis of a Steady-State Method for Estimating Dust Production and Deposition Rates in a Ventilated Airspace

Y. Chen,[1] E. M. Barber,[2] Y. Zhang[3]

[1] Department of Agricultural and Biosystems Engineering, Iowa State University, Ames, IA 50011, USA
[2] Department of Agricultural and Bioresource Engineering, University of Saskatchewan, Saskatoon, Canada, S7H 5N9
[3] Department of Agricultural Engineering, University of Illinois at Urbana-Champaign, Urbana-Champaign, IL 61801, USA

Received: 20 October 1997/Accepted: 18 February 1998

Air quality in livestock buildings can be adversely affected by the presence of atmospheric contaminants. These contaminants include water vapour, carbon dioxide, manure gas such as ammonia and hydrogen sulfide, dust, and viable microorganisms. Of all contaminants, dust is considered to be particularly hazardous because it carries viruses and bacteria to the respiratory systems of livestock (Donham, 1986), and dust inhalation could obstruct the respiratory systems of farm workers (Donham et al., 1988).

In order to control dust contamination and assess the effectiveness of dust control technologies and strategies, dust production and deposition need to be determined (Albright, 1989; Kusuda, 1989; Int-Hout III, 1989). A steady-state method was recently developed by Chen et al. (1997) to measure dust production and deposition rates for a ventilated airspace. The objective of this research was to evaluate this method based on experimental data and different statistical methods.

MATERIALS AND METHODS

Dust concentration for a ventilated airspace under steady-state conditions is as follows (Chen et al., 1997):

$$C = \frac{m_p + Q_s C_s}{Q_s + \eta Q_r + \alpha} \qquad (1)$$

where: C is the dust concentration within the airspace, mg/m^3; C_s is the dust concentration of the supply air, mg/m^3; m_P is the dust production rate, mg/s; Q_s is the air exchange rate through ventilation inlet, m^3/s; Q_r is the air exchange rate through recirculation system, m^3/s; η is the filtration efficiency; and α is the overall dust deposition coefficient, m^3/s.

By employing a change of variable:

$$Y = \frac{1}{C} \qquad (2)$$

Correspondence to: Y. Chen

Equation (1) is transformed to a linear model:

$$Y = a_1 + a_2 \eta \quad (3)$$

where a_1 and a_2 are constants:

$$a_1 = \frac{Q_s + \alpha}{m_p + Q_s C_s} \quad (4)$$

and

$$a_2 = \frac{Q_r}{m_p + Q_s C_s} \quad (5)$$

If C versus η are available, a parameter estimation method could be used to estimate both a_1 and a_2 in Equation (3). The estimates of m_P (β_1) and α (β_2) can then be derived from Equations (4) and (5) as:

$$\beta_1 = \frac{Q_r}{a_2} - Q_s C_s \quad (6)$$

$$\beta_2 = a_1 (\beta_1 + Q_s C_s) - Q_s \quad (7)$$

where β_1 is the estimate of m_P, and β_2 is the estimate of a.

The dust deposition rate can be calculated by:

$$m_d = b_2 C \quad (8)$$

where: m_d is the dust deposition rate, mg/s.

An experiment was conducted to measure airborne dust concentrations at different filtration efficiencies. The experiment was carried out in a large enclosed building airspace where temperature and relative humidity were almost constant. The dust concentration of the supply air was relatively constant and air distribution was in isothermal conditions.

The experimental facilities consisted of an air supply and ventilation measurement system, a recirculation system, a dust production system, a dust measurement system, and a chamber. The dimension of the chamber was 4.88 x 2.66 x 2.06 m (LxWxH). Detailed description of the experimental facilities can be found in Chen and Barber (1997).

Experimental data (Chen et al., 1997) are shown in Table 1. Data in the first two columns of Table 1 were used as inputs to the parameter estimation method, and the data in the rest of the columns were used to validate the parameter estimation results.

Table 1. Experimental data

η	C (mg/m^3)	m_p (mg/s)	α (m^3/s)	m_d (mg/s)
0.0	5.17	3.32	0.4603	2.3796
0.5	4.76	3.32	-	-
1.0	4.38	3.32	-	-

In order to investigate the effect of different parameter estimation methods on the accuracy of final results, two methods were used to estimate the parameters in Equation (3) with the same data: the sequential method and the least squares method. Both methods employ the same principle: minimizing the deviation of the transformed variable. The advantage of the sequential method is that the matrix inversion is not needed because there are no simultaneous equations to solve (Chen et al., 1998). In the least squares method, the sum of squares function (S) to be minimized with respect to the parameters is written as:

$$S = \sum_{i=1}^{n} (a_1 + a_2 \eta_i - Y_i)^2 \tag{9}$$

where S is the sum of squares, Y_i is the observations, *i* is the data point, and n is the total number of measurements. The equations to calculate a_1 and a_2 are available from Kennedy and Neville (1976).

Two approaches were used to analyze the error caused by the linear transformation technique: (a) the weighting method in which the parameters in Equation (3) were estimated by minimizing the deviation of the original variable, and (b) the Marquardt nonlinear method (SAS, 1994) in which the parameters in Equation (1) were directly estimated.

To minimize the deviation of the original variable by the weighting method, it was assumed that all values of C had equal precision. This assumption does not mean that the error of all values l/C is the same. In reality, the values of Y = l/C will not be of equal precision, and weighting considerations will have to be applied.

The variance of Y = 1/C could be expressed as

$$\sigma_Y^{\ 2} = \left(\frac{\partial Y}{\partial C}\right)^2 \sigma_C^{\ 2} = \frac{\sigma_C^{\ 2}}{C^4} \tag{10}$$

where σ_y^2 is the variance of Y, and σ_C^2 is the variance of C.
Then

$$\sigma_Y = \frac{\sigma_C}{C^2} \tag{11}$$

The criterion was that the weight of a value is proportional to $(\text{error})^{-2}$ (Kennedy and Neville, 1976). The weight of each value of Y is proportional to $(\sigma_c / C^2)^{-2}$. Since σ_c is constant, the weight of each value of Y is proportional to C^4.

The sum of squares function for the weighting method can be written as:

$$S = \sum_{i=1}^{n} w_i (a_1 + a_2 \eta_i - Y_i)^2 \tag{12}$$

where w is the weighting factor listed in Table 2.

Table 2. Weighting factors for Y

η	0.0	0.5	1.0
C	5.17	4.76	4.38
Y	0.19	0.21	0.23
w	714.43	513.37	368.04

Differentiating S with respect to a_1, and setting the derivative equal to zero, and repeating the same procedure for a_2, yields:

$$\frac{\partial S}{\partial a_1} = 2\sum_{i=1}^{n} w_i (a_1 + a_2 \eta_i - Y_i) = 0 \tag{13}$$

$$\frac{\partial S}{\partial a_2} = 2\sum_{i=1}^{n} w_i (a_1 + a_2 \eta_i - Y_i)\eta_i = 0 \tag{14}$$

Solving (13) and (14) for a_1 and a_2, gives:

$$a_1 = \frac{\sum_{i=1}^{n} w_i Y_i - a_2 \sum_{i=1}^{n} w_i \eta_i}{\sum^{n} w_i} \tag{15}$$

$$a_2 = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i Y_i \eta_i - \sum_{i=1}^{n} w_i \eta_i \sum_{i=1}^{n} w_i Y_i}{\sum_{i=1}^{n} w_i [\sum_{i=1}^{n} w_i \eta_i^2 - (\sum_{i=1}^{n} w_i \eta_i)^2]} \tag{16}$$

In the Marquardt nonlinear method the parameters are iteratively adjusted to minimize the sum squared function. This nonlinear fitting procedure requires matrix inversion with each iteration (Beck and Arnold, 1977).

RESULTS AND DISCUSSION

Table 3 shows the parameter estimation results by using different parameter estimation methods. Comparison of the results obtained by the three linear methods (Table 4) shows that there is no significant difference ($p < 0.01$). This indicates that the final results of the steady-state method for estimating dust production and deposition rates are not affected by different linear estimation methods.

Table 3. Parameter estimation by different parameter estimation methods

	η	C (mg/m^3)	m_p (mg/s)	α (m^3/s)	m_d (mg/s)
Sequential	0.0	5.1765	3.3483	0.4648	2.4062
	0.5	4.7471	3.3483	-	-
	1.0	4.3836	3.3483	-	-
Least squares	0.0	5.1773	3.3524	0.4655	2.4100
	0.5	4.7483	3.3524	-	-
	1.0	4.3850	3.3524	-	-
Weighting	0.0	5.1753	3.3684	0.4688	2.4265
	0.5	4.7485	3.3684	-	-
	1.0	4.3868	3.3684	-	-
Marquardt	0.0	5.1749	3.3703	0.4693	2.4286
	0.5	4.7484	3.3703	-	-
	1.0	4.3868	3.3703	-	-

Table 4. Comparison of the results by linear methods

	m_p (mg/s)	Error (%)	m_d (mg/s)	Error (%)
Sequential	3.3483	-	2.4062	-
Least squares	3.3524	-012	2.4100	-0.16
Weighting	3.3684	-0.60	2.4265	-0.84

Table 5 shows a comparison of the measured with the estimated results by the linear and nonlinear methods. The relative errors of all of the methods are less than 3 %. The linear parameter estimation method is much easier to use. The differences between the results obtained from the linear methods and the nonlinear method are less than 1 %, proved that the linear transformation technique did not generate extra error in the final results.

Table 5. Comparison of the measured results with the estimated by different methods

	m_p (mg/s)	Error (%)	m_d (mg/s)	Error (%)
Experimental	3.32	-	2.3796	-
Sequential	3.3483	-0.85	2.4062	-1.12
Least squares	3.3524	-0.98	2.4100	-1.28
Weighting	3.3684	-1.46	2.4265	-1.97
Marquardt	3.3702	-1.5	2.4286	-2.06

The sensitivity analysis was conducted in terms of initial values of the parameters for the sequential and the nonlinear Marquardt methods. Table 6 shows the sequential parameter estimation results based on different initial values of β_1 and β_2

Table 6. Sequential estimation based on different initial values of β_1 and β_2

Iteration	β_1	β_2	S
	Initial values β_1=4.0, β_2=0.5		
1	0.2543	-0.1328	13.8733
2	3.5143	0.4978	0.0009
3	3.3483	0.4648	0.0002
	Initial values β_1=2.0, β_2=0.25		
1	0.2340	-0.1367	14.9067
2	3.5154	0.4981	0.0009
3	3.3565	0.4664	0.0002
	Initial values β_1=10.0, β_2=1.0		
1	0.4680	-0.0915	7.1387
2	3.5133	0.4976	0.0009
3	3.3550	0.4661	0.0002
	Initial values β_1=1000.0, β_2=500.0		
1	0.0023	-0.1815	41.2923
2	3.3119	0.4586	0.0006
3	3.3533	0.4668	0.0004

From Table 6, the final results do not change significantly for different initial values of parameters (the maximum relative error < 1 %). When using the sequential method, the user does not have to have the knowledge or information about the possible range of the parameters. This conclusion is also held for the Marquardt method.

Table 7. Marquardt estimation based on different initial values of β_1 and β_2

Iteration	β_1	β_2	S
0	4.0	0.5	1.2872
1	3.6931	0.5314	0.0052
2	3.4604	0.4882	0.0004
3	3.3718	0.4696	0.0002
4	3.3703	0.4693	0.0002
5	3.3703	0.4693	0.0002
0	10.0000	1.0000	32.7347
1	7.5226	1.1955	0.7411
2	4.5586	0.7590	0.1507
3	2.9479	0.3641	0.0697
4	3.2947	0.4510	0.0014
5	3.3678	0.4687	0.0002
6	3.3703	0.4693	0.0002
7	3.3703	0.4693	0.0002
0	20.0000	5.0000	2.9746
1	21.8418	4.1753	0.3111
2	15.3286	3.0129	0.2025
3	10.0000	1.8394	0.1399
4	5.9392	1.0106	0.0603
5	4.0299	0.6042	0.0088
6	3.2913	0.4532	0.0004
7	3.3684	0.4689	0.0002
8	3.3703	0.4693	0.0002
9	3.3703	0.4693	0.0002

Table 7 shows the effect of different initial values of parameters on convergence speed and final results for the Marquardt method. Selection of initial values of the parameters will not affect the final results. However, a close-to-final-result guess of the initial values will yield fast convergence.

REFERENCES

Albright LD (1989) Research needs. In: Building Systems: Room Air and Air Contaminant Distribution, 8-9. Atlanta: ASHRAE

Barber EM (1991) A review of methods for estimating the role of dust deposition in removal of dust from livestock building airspaces. ASAE Paper No. 91-4551. St. Joseph, MI: ASAE

Beck JV, Arnold KJ (1977) Parameter estimation in engineering and science. John Wiley & Sons, New York

Chen Y C, Barber EM (1997) An experimental facility for research on dust generation and deposition rates. J. Environ Sci Health 32: In press
Chen YC, Barber EM, Zhang Y, Besant R, Sohkasanj S (1997) Methods to measure dust production and deposition rates in buildings. J. Agricultural Engineering Research. Accepted
Chen YC, Barber EM, Zhang Y (1998) A method to measure dust production and deposition rates for a ventilated airspace without recirculation systems. ASHARE Transactions. 104. In press
Donham KJ (1986) Hazardous agents in agricultural dusts and methods of evaluation. American Journal of Industrial Medicine 10: 205-220
Donham KJ, Haglind P, Peterson Y, Rylander R, Belin L (1988) Environmental and health studies of workers in Swedish swine confinement buildings. British Journal of Industrial Medicine 40: 31-37
Int-Hout III D (1989) Physical Modelling. In: Building Systems: Room Air and Air Contaminant Distribution, 22-23. Atlanta: ASHRAE
Kennedy JB, Neville AM (1976) Basic statistical methods for engineers and scientist. Harper & Row Publishers, New York
Kusuda T (1989) Mathematical modeling. In: Building Systems: Room Air and Air Contaminant Distribution, 17-21. Atlanta: ASHRAE
SAS. 1994. Operating manual. SAS Institute, Inc, Cary